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SPG-Separation Axioms

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Abstract

In this paper we discuss new separation axioms using spg-open sets

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Introduction

Norman Levine introduced generalized closed sets in 1970. After him various Authors[1-18; 20-29] studied different versions of generalized sets and related topological properties. Recently V.K. Sharma and the author of the present paper defined separation axioms for g-open; gs-open; sg-open; rg-open sets and studied their basic properties.. Throughout the paper a space X means a topological space (X,τ). For any subset A of X its complement, interior, closure, spg-interior, spg-closure are denoted respectively by the symbols A^c, A^o, cl(A), spg-int(A) and spg-cl(A).

Definition 1.1: $A \subseteq X$ is called

(i) regularly open if A = int(cl(A)) and regularly closed if A = cl(int(A)).

(ii) semi-open if there exists an open set U such that $U\subseteq A\subseteq cl(U)$.

(iii) generalized closed[resp: regular generalized; generalized regular]{briefly: g-closed; rg-closed; pg-closed}if $cl\{A\}\subseteq U$ whenever $A\subseteq U$ and U is open[resp: regular open, open] and generalized[resp: regular generalized; generalized regular] open if its complement is generalized[resp: regular generalized; generalized regular] closed.

Note 1: The class of regular open sets, open sets, g-open sets and spg-open sets are denoted by RO(X), $\tau(X)$, GO(X) and SPGO(X) respectively. Clearly $RO(X)\subset\tau(X)\subset GO(X)\subset PGO(X)$.

Note 2: For $A\subset X$, $A\in PGO(X, x)$ means A is a generalized regular-open neighborhood of X containing x.

Definition 1.3: $A\subset X$ is called clopen[resp: nearly-clopen; semi-clopen; g-clopen; spg-clopen] if it is both open[resp: regular-open; semi-open; g-open; spg-open] and closed[resp: regular-closed; semi-closed; g- closed; spg-closed]

Definition 1.4: A function $f: X \to Y$ is said to be

(i) Continuous [resp: nearly continuous, semi-continuous] if inverse image of open set is open[resp: regular-open, semi-open]

(ii) g-continuous [resp: spg-continuous] if inverse image of closed set is g-closed [resp: spg-closed]

(iii) irresolute [resp: nearly irresolute, spg-irresolute] if inverse image of semi-open [resp: regular-open, spg-open] set is semi-open [resp: regular-open, spg-open]

(iv) g-irresolute [resp: spg-irresolute; sg-irresolute] if inverse image of g-closed [resp: spg-closed, sg-closed] set is g-closed [resp: spg-closed; sg-closed]

(v) open [resp: nearly open, semi-open] if the image of open set is open [resp: regular-open, semi-open]

(vi) g-open [resp: spg-open] if the image of open set is g-open [resp: spg-open]

(vii) homeomorphism [resp: nearly homeomorphism, semi-homeomorphism] if f is bijective, continuous [resp: nearly-continuous, semi-continuous] and f^{-1} is continuous[resp: nearly-continuous, semi-continuous]

(viii) rc-homeomorphism [resp: sc-homeomorphism] if f is bijective r-irresolute [resp: irresolute] and f^{1} is r-irresolute [resp: irresolute]

(ix) g-homeomorphism [resp: spg-homeomorphism] if f is bijective g-continuous [resp: spg-continuous] and f^{-1} is g-continuous [resp: spg-continuous]

(x) gc-homeomorphism [resp: spgc-homeomorphism] if f is bijective g-irresolute [resp: spg-irresolute] and f^{-1} is g-irresolute[resp: spg-irresolute]

Definition 1.5: X is said to be
(i) compact [resp: nearly compact, semi-compact, g-compact, spg-compact] if every open[resp: regular-open, semi-open, g-open, spg-open] cover has a finite sub cover.
(ii) T_0 [resp: rT_0, sT_0, g_0] space if for each $x \neq y \in X$ $\exists$ $U \in \tau(X)$[resp: RO(X); SO(X); GO(X)] containing either x or y.
(iii) T_1 [resp: rT_1, sT_1, g_1] space if for each $x \neq y \in X$ $\exists$ U, $V \in \tau(X)$[resp: RO(X); SO(X); GO(X)] such that $x \in U - V$ and $y \in V - U$.
(iv) T_2 [resp: rT_2, sT_2, g_2] space if for each $x \neq y \in X$ $\exists$ U, $V \in \tau(X)$[resp: RO(X); SO(X); GO(X)] such that $x \in U$; $y \in V$ and $U \cap V = \phi$.
(v) $T_{1/2}$ [resp: $rT_{1/2}$, $pT_{1/2}$] if every generalized [resp: regular generalized, pre-generalized] closed set is closed [resp: regular-closed, pre-closed]

Spg-Continuity and Product Spaces

Theorem 2.1: Let Y and $\{X_\alpha : \alpha \in I\}$ be Topological Spaces. Let $f: Y \to \Pi X_\alpha$ be a function. If f is spg-continuous, then $\pi_\alpha \bullet f: Y \to X_\alpha$ is spg-continuous.

Proof: Suppose f is spg-continuous. Since $\pi_\alpha: \Pi X_\beta \to X_\alpha$ is continuous for each $\alpha \in I$, it follows that $\pi_\alpha \bullet f$ is spg-continuous.

Converse of the above theorem is not true in general.

Theorem 2.2: If Y is $rT_{1/2}$ and $\{X_\alpha : \alpha \in I\}$ be Topological Spaces. Let $f: Y \to \Pi X_\alpha$ be a function, then f is spg-continuous iff $\pi_\alpha \bullet f: Y \to X_\alpha$ is spg-continuous.

Corollary 2.3: Let $f_\alpha: X_\alpha \to Y_\alpha$ be a function and let $f: \Pi X_\alpha \to \Pi Y_\alpha$ be defined by $f(x_\alpha)_{\alpha \in I} = (f_\alpha(x_\alpha))_{\alpha \in I}$. If f is spg-continuous then each f_α is spg-continuous.

Corollary 2.4: For each α, let X_α be $rT_{1/2}$ and let $f_\alpha: X_\alpha \to Y_\alpha$ be a function and let $f: \Pi X_\alpha \to \Pi Y_\alpha$ be defined by $f(x_\alpha)_{\alpha \in I} = (f_\alpha(x_\alpha))_{\alpha \in I}$, then f is spg-continuous iff each f_α is spg-continuous.

Spg$_i$ Spaces i = 0, 1, 2

Definition 3.1: X is said to be
(i) a spg_0 space if for each pair of distinct points x, y of X, there exists a spg-open set G containing either of the point x or y.
(ii) a spg_1 space if for each pair of distinct points x, y of X there exists a spg-open set G containing x but not y and a spg-open set H containing y but not x.
(iii)a spg_2 space if for each pair of distinct points x, y of X there exists disjoint spg-open sets G and H such that G containing x but not y and H containing y but not x.

Note 2: X is $spg_2 \to$ X is $spg_1 \to$ X is spg_0.

Example 3.1: Let X = {a, b, c} and
(i) $\tau = \{\phi, \{a, c\}, X\}$ then X is spg_i but not rT_0 and T_0, i = 0, 1, 2.
(ii) $\tau = \{\phi, \{a\}, \{a, c\}, X\}$ then X is not spg_i for i = 0, 1, 2.

Remark 3.1: If X is $pT_{1/2}$ then pT_i and spg_i are one and the same for i = 0,1,2.

Theorem 3.1:
(i) Every [resp: regular open] open subspace of spg_i space is spg_i for i = 0, 1, 2.
(ii)The product of spg_i spaces is again spg_i for i = 0, 1, 2.
(iii) spg-continuous image of T_i spaces is spg_i for i = 0, 1, 2.
(iv) spg-continuous image of rT_i spaces is spg_i for i = 0, 1, 2.

Theorem 3.2:
(i) X is spg_0 iff $\forall$ $x \in X$, $\exists$ $U \in SPGO(X)$ containing x such that the subspace U is spg_0.
(ii)X is spg_0 iff distinct points of X have disjoint spg-closures.

Theorem 3.3: The following are equivalent:
(i) X is spg_1.
(ii) Each one point set is spg-closed.
(iii)Each subset of X is the intersection of all spg-open sets containing it.
(iv) For any $x \in X$, the intersection of all spg-open sets containing the point is the set {x}.

Theorem 3.4: If X is spg_1 then distinct points of X have disjoint spg-closures.

Theorem 3.5: Suppose x is a spg-limit point of a subset of A of a spg_1 space X. Then every neighborhood of x contains infinitely many distinct points of A.

Theorem 3.6: X is spg_2 iff the intersection of all spg-closed, spg-neighborhoods of each point of the space is reduced to that point.

Proof: Let X be spg_2 and $x \in X$, then for each $y \neq x$ in X, $\exists$ U, $V \in SPGO(X)$ such that $x \in U$, $y \in V$ and $U \cap V = \phi$. Since $x \in U - V$, hence X-V is a spg-closed, spg-neighborhood of x to which y does not belong. Consequently, the intersection of all spg-closed, spg-neighborhoods of x is reduced to {x}.

Conversely let $y \neq x$ in X, then by hypothesis there exists a spg-closed, spg-neighborhood U of x such that $y \notin U$. Now $\exists$ $G \in SPGO(X)$ such that $x \in G \subset U$. Thus G and X-U are disjoint spg-open sets containing x and y respectively. Hence X is spg_2.

Theorem 3.7: If to each point $x\in X$, there exist a spg-closed, spg-open subset of X containing x which is also a spg_2 subspace of X, then X is spg_2.
Proof: Let $x\in X$, U a spg-closed, spg-open subset of X containing x and which is also a spg_2 subspace of X, then the intersection of all spg-closed, spg-neighborhoods of x in U is reduced to {x}. U being spg-closed, spg-open, these are spg-closed, spg-neighborhoods of x in X. Thus the intersection of all spg-closed, spg-neighborhoods of x is reduced to {x}. Hence by Theorem 3.6, X is spg_2.

Theorem 3.8: If X is spg_2 then the diagonal Δ in $X\times X$ is spg-closed.
Proof: Let $(x, y)\in X\times X-\Delta$, then $x \neq y$. Since X is spg_2 $\exists$ U; $V\in SPGO(X)$ such that $x\in U$; $y\in V$ and $U\cap V = \phi$. $U\cap V = \phi$ implies $(U\times V)\cap\Delta = \phi$ and therefore $(U\times V)\subset X\times X-\Delta$. Further $(x, y)\in (U\times V)$ and $(U\times V)$ is spg-open in $X\times X$ gives $X\times X-\Delta$ is spg-open. Hence Δ is spg-closed.

Theorem 3.9: In spg_2-space, spg-limits of sequences, if exists, are unique.

Theorem 3.10: In a spg_2 space, a point and disjoint spg-compact subspace can be separated by disjoint spg-open sets.
Proof: Let X be a spg_2 space, $x\in X$ and C a spg-compact subspace of X not containing x. Let $y\in C$ then for $x \neq y$ in X, there exist disjoint spg-open neighborhoods G_x and H_y. Allowing this for each y in C, we obtain a class $\{H_y\}$ whose union covers C; and since C is spg-compact, some finite subclass $\{H_i, i = 1 \text{ to } n\}$ covers C. If G_i is spg-neighborhood of x corresponding to H_i, we put $G = \cup_{i=1-n}G_i$ and $H = \cap_{i=1-n}H_i$, satisfying the required properties.

Corollary 3.1:
(i) In a T_1 [resp: rT_1; g_1] space, each singleton set is spg-closed.
(ii) If X is T_1 [resp: rT_1; g_1] then distinct points of X have disjoint spg-closures.
(iii)If X is T_2 [resp: rT_2; g_2] then the diagonal Δin $X\times X$ is spg-closed.
(iv) Show that in a T_2 [resp: rT_2; g_2] space, a point and disjoint compact[resp: nearly-compact; g-compact] subspace can be separated by disjoint spg-open sets

Theorem 3.11: Every spg-compact subspace of a spg_2 space is spg-closed.
Proof: Let C be spg-compact subspace of a spg_2 space. If x be any point in C^c, by above Theorem x has a spg-neighborhood G such that $x\in G \subset C^c$. This shows that C^c is the union of spg-open sets and therefore C^c is spg-open. Thus C is spg-closed.

Corollary 3.2: Every compact [resp: nearly-compact; g-compact] subspace of a T_2 [resp: rT_2; g_2] space is spg-closed.

Theorem 3.12: If *f:* $X\rightarrow Y$ is injective, spg-irresolute and Y is spg_i then X is spg_i, i = 0, 1, 2.
Proof: Let $x \neq y\in X$, then $\exists$ a spg-open set $V_x\subset Y$ such that $f(x)\in V_x$ and $f(y)\notin V_x$ and $\exists$ a spg-open set $V_y\subset Y$ such that $f(y)\in V_y$ and $f(x)\notin V_y$ with $f(x) \neq f(y)$. By spg-irresoluteness of *f*, $f^{-1}(V_x)$ is spg-open in X such that $x\in f^{-1}(V_x)$; $y\notin f^{-1}(V_x)$ and $f^{-1}(V_y)$ is spg-open in X such that $y\in f^{-1}(V_y)$; $x\notin f^{-1}(V_y)$. Hence X is spg_2

Similarly one can prove the remaining part of the Theorem.

Corollary 3.3:
(i) If *f:* $X\rightarrow Y$ is injective, spg-continuous and Y is T_i then X is spg_i, i = 0, 1, 2.
(ii) If *f:* $X\rightarrow Y$ is injective, r-irresolute[r-continuous] and Y is rT_i then X is spg_i, i = 0, 1, 2.
(iii)The property of being a space is spg_0 is a spg-Topological property.
(iv) Let *f:* $X \rightarrow Y$ is a spgc-homeomorphism, then X is spg_i if Y is spg_i, i = 0, 1, 2.

Theorem 3.13: Let X be T_1 and *f*: $X \rightarrow Y$ be spg-closed surjection. Then X is spg_1.

Theorem 3.14: Every spg-irresolute map from a spg-compact space into a spg_2 space is spg-closed.
Proof: If *f*: $X \rightarrow Y$ is spg-irresolute where X is spg-compact and Y is spg_2. Let $C\subset X$ be closed, then $C\subset X$ is spg-closed and hence C is spg-compact and so $f(C)$ is spg-compact. But then $f(C)$ is spg-closed in Y. Hence the image of any spg-closed set in X is spg-closed set in Y. Thus *f* is spg-closed.

Theorem 3.15: Any spg-irresolute bijection from a spg-compact space onto a spg_2 space is a spgc-homeomorphism.
Proof: Let *f*: $X \rightarrow Y$ be a spg-irresolute bijection from a spg-compact space onto a spg_2 space. Let G be a spg-open subset of X. Then X-G is spg-closed and hence $f(X-G)$ is spg-closed. Since *f* is bijective $f(X-G) = Y-f(G)$. Therefore $f(G)$ is spg-open in Y. This means that *f* is spg-open. Hence *f* is bijective spg-irresolute and spg-open. Thus *f* is spgc-homeomorphism.

Corollary 3.4: Any spg-continuous bijection from a spg-compact space onto a spg_2 space is a spg-homeomorphism.

Theorem 3.16: The following are equivalent:
(i) X is spg_2.
(ii) For each pair $x \neq y \in X$ $\exists$ a spg-open, spg-closed set V such that $x \in V$ and $y \notin V$, and
(iii)For each pair $x \neq y \in X$ $\exists$ $f: X \to [0, 1]$ such that $f(x) = 0$ and $f(y) = 1$ and f is spg-continuous.

Theorem 3.17: If $f: X \to Y$ is spg-irresolute and Y is spg_2 then
(i) the set $A = \{(x_1, x_2): f(x_1) = f(x_2)\}$ is spg-closed in $X \times X$.
(ii)G(f), Spgaph of f, is spg-closed in $X \times Y$.
Proof: (i) Let $A = \{(x_1, x_2): f(x_1) = f(x_2)\}$. If $(x_1, x_2) \in X \times X\text{-}A$, then $f(x_1) \neq f(x_2) \Rightarrow \exists$ disjoint V_1 and $V_2 \in SPGO(Y)$ such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$, then by spg-irresoluteness of f, $f^{-1}(V_j) \in SPGO(X, x_j)$ for each j. Thus $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \in SPGO(X \times X)$. Therefore $f^{-1}(V_1) \times f^{-1}(V_2) \subset X \times X\text{-}A \Rightarrow X \times X\text{-}A$ is spg-open. Hence A is spg-closed.

(ii) Let $(x, y) \notin G(f) \Rightarrow y \neq f(x) \Rightarrow \exists$ disjoint spg-open sets V and W such that $f(x) \in V$ and $y \in W$. Since f is spg-irresolute, $\exists$ $U \in SPGO(X)$ such that $x \in U$ and $f(U) \subset W$. Therefore we obtain $(x, y) \in U \times V \subset X \times Y$, where $U \times V \subset X \times Y\text{-}G(f)$. Hence $X \times Y\text{-}G(f)$ is spg-open. Hence $G(f)$ is spg-closed in $X \times Y$.

Theorem 3.18: If f: $X \to Y$ is spg-open and $A = \{(x_1, x_2): f(x_1) = f(x_2)\}$ is closed in $X \times X$. Then Y is spg_2.

Theorem 3.19: Let Y and $\{X_\alpha : \alpha \in I\}$ be Topological Spaces. If $f: Y \to \Pi X_\alpha$ be a spg-continuous function and Y is $rT_{1/2}$, then ΠX_α and each X_α are spg_i, i = 0,1,2.

Problem: If Y be a spg_2 space and A be regular-open subspace of X. If f: $(A, \tau_{/A}) \to (Y, \sigma)$ is spg-irresolute. Is there exists any extension f: $(X, \tau) \to (Y, \sigma)$.

Theorem 3.20: Let X be an arbitrary space, R an equivalence relation in X and p: $X \to X/R$ the identification map. If $R \subset X \times X$ is spg-closed in $X \times X$ and p is spg-open map, then X/R is spg_2.
Proof: Let $p(X)$, $p(y)$ be distinct members of X/R. Since x and y are not related, $R \subset X \times X$ is spg-closed in $X \times X$. There are spg-open sets U and V such that $x \in U$, $y \in V$ and $U \times V \subset R^c$. Thus $\{p(U), p(V)\}$ are disjoint and also spg-open in X/R since p is spg-open.

Theorem 3.21: The following four properties are equivalent:
(i) X is spg_2
(ii) Let $x \in X$. For each $y \neq x$, $\exists$ $U \in SPGO(X)$ such that $x \in U$ and $y \notin spgcl(U)$
(iii)For each $x \in X$, $\cap\{spgcl(U)/U \in SPGO(X)$ and $x \in U\} = \{x\}$.
(iv) The diagonal $\Delta = \{(x, x)/x \in X\}$ is spg-closed in $X \times X$.
Proof: (i) $\Rightarrow$ (ii) Let $x \in X$ and $y \neq x$. Then there are disjoint spg-open sets U and V such that $x \in U$ and $y \in V$. Clearly V^c is spg-closed, $spgcl(U) \subset V^c$, $y \notin V^c$ and therefore $y \notin spgcl(U)$.
(ii) $\Rightarrow$ (iii) If $y \neq x$, then $\exists$ $U \in SPGO(X)$ s.t. $x \in U$ and $y \notin spgcl(U)$. So $y \notin \cap\{spgcl(U)/U \in SPGO(X)$ and $x \in U\}$.
(iii) $\Rightarrow$ (iv) We prove Δ^c is spg-open. Let $(x, y) \notin \Delta$. Then $y \neq x$ and $\cap\{spgcl(U)/U \in SPGO(X)$ and $x \in U\} = \{x\}$ there is some $U \in SPGO(X)$ with $x \in U$ and $y \notin spgcl(U)$. Since $U \cap (spgcl(U))^c = \phi$, $U \times (spgcl(U))^c$ is a spg-open set such that $(x, y) \in U \times (spgcl(U))^c \subset \Delta^c$.
(iv) $\Rightarrow$ (i) $y \neq x$, then $(x, y) \notin \Delta$ and thus there exist spg-open sets U and V such that $(x, y) \in U \times V$ and $(U \times V) \cap \Delta = \phi$. Clearly, for the spg-open sets U and V we have; $x \in U$, $y \in V$ and $U \cap V = \phi$.

$Spgg_3$ and $Spgg_4$ spaces

Definition 4.1: X is said to be
(i) a spg_3 space if for every spg-closed sets F and a point $x \notin F$ $\exists$ disjoint U, $V \in PO(X)$such that $F \subseteq U$; $x \in V$
(ii) a $spgg_3$ space if for every spg-closed sets F and $x \notin F$ $\exists$ disjoint U, $V \in SPGO(X)$such that $F \subseteq U$; $x \in V$
(iii)a spg_4 space if for each pair of disjoint spg-closed sets F and H $\exists$ disjoint U, $V \in PO(X)$ s.t. $F \subseteq U$; $H \subseteq V$
(iv) a $spgg_4$ space if for each pair of disjoint spg-closed sets F and H $\exists$ disjoint U, $V \in SPGO(X)$ s.t. $F \subseteq U$; $H \subseteq V$

Note: $rT_i \to spg_i \to spgg_i$, i = 3, 4. but the converse is not true in general.

Example 4.1: Let X = {a, b, c} and
(i) $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ then X is $spgg_i$.
(ii) $\tau = \{\phi, \{a\}, X\}$ then X is not $spgg_i$, spg_i and rT_i for i = 3, 4.

Lemma 4.1: X is spg-regular iff X is nearly-regular and $rT_{1/2}$.
Proof: X is spg-regular, then obviously X is nearly-regular. Let $A \subseteq X$ be spg-closed. For each $x \notin A$ $\exists$ $V_x \in SPGO(X, x)$ such that $V_x \cap A = \phi$. If $V = \cup\{V_x : x \notin A\}$, then V is spg-open and V = X-A. Hence A is spg-closed implies X is $rT_{1/2}$.

Theorem 4.1: If X is spg_3. Then for each x∈X and each U∈SPGO(X, x) ∃ a spg-neighborhood V of x such that spgcl(A)⊂U.

Proof: Let x∈X and U a spg-neighborhood of x. Let B = X - U, then B is spg-closed and by spg-regularity of X, ∃ disjoint V, W∈SPGO(X) such that x∈V and B⊆W. Then spgcl(V)∩B = ϕ ⇒ spgcl(V)⊆X - B.

Theorem 4.2: The following are equivalent:

(i) X is spg_3.

(ii) For every point x∈X and for every G∈SPGO(X, x), ∃ U∈SPGO(X) such that x∈U⊆ spgcl(U) ⊆ G.

(iii)For every spg-closed set F, the intersection of al spg-closed spg-neighborhoods of F is exactly F.

(iv) For every set A and B∈SPGO(X) such that A∩B ≠ ϕ, ∃ G∈SPGO(X)such that A∩G ≠ ϕ and spgcl(G)⊆B.

(v) For every A ≠ ϕ and B∈SPGC(X) with A∩B = ϕ, ∃ disjoint G; H∈SPGO(X) such that A⊆G and B⊆H.

Theorem 4.3: If X is $spgg_3$. Then for each x∈X and each U∈SPGO(X, x), ∃ V∈SPGO(X, x) such that spgcl(A)⊂U.

Proof: Let x∈ X and U a spg-neighborhood of x. Let B = X - U, then B is spg-closed and by spgg-regularity of X, ∃ disjoint V, W∈SPGO(X) such that x∈V and B⊆W. Then spgcl(V)∩B = ϕ ⇒ spgcl(V)⊆X - B.

Corollary 4.1: If X is T_3 [resp: rT_3; g_3]. Then for each x∈ X and each spg-open neighborhood U of x there exists a spg-open neighborhood V of x such that spgcl(A)⊂U.

Theorem 4.4: If *f:* X→ Y is spg-closed, spg-irresolute bijection. Then X is $spgg_3$ iff Y is $spgg_3$.

Proof: Let F be closed set in X and x∉F, then *f*(x)∉*f*(F) and *f*(F) is spg-closed in Y. By $spgg_3$ of Y, ∃ V; W∈ SPGO(y) such that *f*(X)∈V and *f*(F)⊆W. Hence x∈f^{-1}(V) and F⊆f^{-1}(W), where f^{-1}(V) and f^{-1}(W) are disjoint spg-open sets in X (by spg-irresoluteness of *f*). Hence X is $spgg_3$.

Conversely, X be $spgg_3$ and K any spg-closed in Y with y∉K, then f^{-1}(K) is spg-closed in X such that f^{-1}(y) ∉ f^{-1}(K). By $spgg_3$ of X, ∃ disjoint V, W∈SPGO(X) such that f^{-1}(y)∈V and f^{-1}(K)⊆W. Hence y∈ *f*(V) and K⊆ *f*(W) such that *f*(V) and *f*(W) are disjoint spg-open sets in X. Thus Y is $spgg_3$

Theorem 4.5: X is spg-normal iff for every spg-closed set F and a spg-open set G containing A, there exists a spg-open set V such that F⊆ V⊆ spgcl(V) ⊆ G

Theorem 4.6: X is spg-normal iff for every pair of disjoint spg-closed sets A and B, there exist disjoint spg-open sets U and V such that A⊆U and B⊆V.

Proof: Necessity: Follows from the fact that every spg-open set is spg-open.

Sufficiency: Let A, B be are disjoint spg-closed sets and U, V are disjoint spg-open sets such that A⊆U and B⊆V. Since U and V are spg-open sets, A⊆U and B⊆V ⇒ A⊆spg$(U)^o$ and B⊆spg$(V)^o$. Hence spg$(U)^o$ and spg$(V)^o$ are disjoint spg-open sets satisfying the axiom of spg-normality.

Theorem 4.7: The following are equivalent:

(i) X is spg-normal

(ii) For any pair of disjoint closed sets A and B, ∃ disjoint U; V∈SPGO(X) such that A⊆ U and B⊆ V

(iii)For every closed set A and an open B containing A, ∃ U∈SPGO(X) such that A⊆ U⊆ spgcl(U) ⊆ B

(iv) For every closed set A and a spg-open B containing A, ∃ U∈SPGO(X) such that A⊆ U⊆ spgcl(U) ⊆ $(B)^o$

(v) For every spg-closed set A and every open B containing A, ∃ U∈SPGO(X) such that A⊆spgcl(A)⊆U⊆spgcl(U) ⊆ B.

Theorem 4.8: The following are equivalent:

(i) X is spg-normal

(ii) For every A∈SPGC(X) and every spg-open set containing A, there exists a spg-clopen set V such that A⊆V⊆U.

Theorem 4.9: Let X be an almost normal space and F∩A = ϕ where F is regularly closed and A is spg-closed, then ∃ disjoint U; V∈τ such that F⊆U; B⊆V.

Theorem 4.10: X is almost normal iff for every disjoint sets F and A where F is regular closed and A is closed, ∃ disjoint spg-open sets in X such that F⊆U; B⊆V.

Proof: Necessity: Follows from the fact that every open set is spg-open.

Sufficiency: Let F, A be disjoint regular closed set F and a closed set A, ∃ disjoint spg-open sets in X such that F⊆U; B⊆V. Hence F⊆U^o; B⊆V^o, where U^o and V^o are disjoint open sets. Hence X is almost regular.

Theorem 4.11: The following are equivalent:

(i) X is almost normal.

(ii) For every regular closed set A and for every spg-open set B containing A, ∃ U∈τ s.t. A⊆U⊆cl(U)⊆ B.

(iii)For every spg-closed set A and for every regular-open set B containing A, ∃ U∈τ s.t. A⊆U⊆cl(U)⊆ B.

(iv) For every pair of disjoint regularly closed set A and spg-closed set B, ∃ U; V∈τ s.t. cl(U)∩cl(V) = ϕ.

Spg-R_i spaces; i = 0,1

Definition 5.1: Let x∈ X. Then
(i) spg-kernel of x is defined and denoted by $Ker_{\{spg\}}\{x\}$ = ∩{U:U∈ SPGO(X) and x∈ U}
(ii)$Ker_{\{spg\}}F$ = ∩{U: U∈ SPGO(X) and F⊂ U}

Lemma 5.1: Let A⊂ X, then $Ker_{\{spg\}}\{A\}$ = {x∈ X: spgcl{x}∩ A ≠ ϕ.}

Lemma 5.2: Let x∈ X. Then y∈ $Ker_{\{spg\}}\{x\}$ iff x∈ spgcl{y}.
Proof: Suppose that y∉$Ker_{\{spg\}}\{x\}$. Then ∃ V∈SPGO(X) containing x such that y∉V. Therefore we have x∉spgcl{y}. The proof of converse part can be done similarly.

Lemma 5.3: For any points x ≠ y∈X, the following are equivalent:
(i) $Ker_{\{spg\}}\{x\}$ ≠ $Ker_{\{spg\}}\{y\}$; (ii) spgcl{x} ≠ spgcl{y}.
Proof: (i) ⇒ (ii): Let $Ker_{\{spg\}}\{x\}$ ≠ $Ker_{\{spg\}}\{y\}$, then ∃ z∈X such that z∈$Ker_{\{spg\}}\{x\}$ and z∉$Ker_{\{spg\}}\{y\}$. From z∈$Ker_{\{spg\}}\{x\}$ it follows that {x}∩spgcl{z} ≠ ϕ ⇒ x∈spgcl{z}. By z∉$Ker_{\{spg\}}\{y\}$, we have {y}∩spgcl{z} = ϕ. Since x∈spgcl{z}, spgcl{x}⊂spgcl{z} and {y}∩spgcl{x} = ϕ. Therefore spgcl{x} ≠ spgcl{y}. Now $Ker_{\{spg\}}\{x\}$ ≠ $Ker_{\{spg\}}\{y\}$ ⇒ spgcl{x} ≠ spgcl{y}.
(ii) ⇒ (i): If spgcl{x} ≠ spgcl{y}. Then ∃ z∈X such that z∈spgcl{x} and z∉spgcl{y}. Then ∃ a spg-open set containing z and therefore containing x but not y, namely, y∉ $Ker_{\{spg\}}\{x\}$. Hence $Ker_{\{spg\}}\{x\}$ ≠ $Ker_{\{spg\}}\{y\}$.

Definition 5.2: X is said to be
(i) spg-R_0 iff spgcl{x}⊆ G whenever x∈ G∈ SPGO(X).
(ii) weakly spg-R_0 iff ∩ spgcl{x} = ϕ.
(iii) spg-R_1 iff for x,y∈X such that spgcl{x} ≠ spgcl{y} ∃ disjoint U; V∈SPGO(X) such that spgcl{x}⊆U and spgcl{y}⊆V.

Example 5.1: Let X = {a, b, c} and τ = {ϕ, {a}, {b, c}, X} then X is spgR_0.

Remark 5.1:
(i) r-R_i ⇒ R_i ⇒ g R_i ⇒ spgR_i, i = 0, 1.
(ii) Every weakly-R_0 space is weakly spg R_0.

Lemma 5.1: Every spgR_0 space is weakly spgR_0.

Converse of the above Theorem is not true in general by the following Examples.

Example 5.2: Let X = {a, b, c, d} and τ = {ϕ, {b}, {a, b}, {b, c}, {a, b, c}, X}, then X is weakly spgR_0 but not spgR_i, i = 0 ,1.

Theorem 5.1: Every spg-regular space X is spg_2 and spg-R_0.
Proof: Let X be spg-regular and let x ≠ y∈ X. By Lemma 4.1, {x} is either spg-open or spg-closed. If {x} is spg-open, {x} is spg-open and hence spg-clopen. Thus {x} and X - {x} are separating spg-open sets. Similar argument, for {x} is spg-closed gives {x} and X - {x} are separating spg-closed sets. Thus X is spg_2 and spg-R_0.

Theorem 5.2: X is spg-R_0 iff given x ≠ y∈ X; spgcl{x} ≠ spgcl{y}.
Proof: Let X be spg-R_0 and let let x, ≠ y∈ X. Suppose U is a spg-open set containing x but not y, then y∈ spgcl{y}⊂ X-U and so x∉ spgcl{y}. Hence spgcl{x} ≠ spgcl{y}.

Conversely, let x, ≠ y∈ X such that spgcl{x} ≠ spgcl{y}⇒ spgcl{x}⊂ X-spgcl{y} = U(say) a spg-open set in X. This is true for every spgcl{x}. Thus ∩ spgcl{x}⊆ U where x∈ spgcl{x}⊆ U∈ SPGO(X), which in turn implies ∩ spgcl{x}⊆ U where x∈ U∈ SPGO(X). Hence X is spgR_0.

Theorem 5.3: X is weakly spgR_0 iff $Ker_{\{spg\}}\{x\}$ ≠ X for any x∈X.
Proof: Let x_0∈ X such that $ker_{\{spg\}}\{x_0\}$ = X. This means that x_0 is not contained in any proper spg-open subset of X. Thus x_0 belongs to the spg-closure of every singleton set. Hence x_0∈∩spgcl{x}, a contradiction.

Conversely assume $Ker_{\{spg\}}\{x\}$ ≠ X for any x∈ X. If there is an x_0∈ X such that x_0∈∩{spgcl{x}}, then every spg-open set containing x_0 must contain every point of X. Therefore, the unique spg-open set containing x_0 is X. Hence $Ker_{\{spg\}}\{x_0\}$ = X, which is a contradiction. Thus X is weakly spg-R_0.

Theorem 5.4: The following statements are equivalent:
(i) X is spg-R_0 space.
(ii) For each x∈ X, spgcl{x}⊂ $Ker_{\{spg\}}\{x\}$
(iii)For any spg-closed set F and a point x∉ F, ∃ U∈ SPGO(X) such that x∉U and F⊂U.
(iv) Each spg-closed set F can be expressed as F = ∩{G: G is spg-open and F⊂G}.

(v) Each spg-open set G can be expressed as G = $\cup\{A$: A is spg-closed and $A\subset G\}$.
(vi) For each spg-closed set F, $x\notin F$ implies spg-cl$\{x\}\cap F = \phi$.

Proof: (i) ⇒ (ii) For any $x\in X$, we have $Ker_{\{spg\}}\{x\} = \cap\{U: U\in SPGO(X)$ and $x\in U\}$. Since X is spg-R_0, each spg-open set containing x contains spgcl{x}. Hence spgcl$\{x\}\subset Ker_{\{spg\}}\{x\}$.

(ii) ⇒ (iii) Let $x\notin F\in SPGC(X)$. Then for any $y\in F$; spgcl$\{y\}\subset F$ and so $x\notin$ spgcl$\{y\}\Rightarrow y\notin$ spgcl{x} that is $\exists\ U_y\in SPGO(X)$ such that $y\in U_y$ and $x\notin U_y\ \forall\ y\in F$. Let $U = \cup\{U_y$: U_y is spg-open, $y\in U_y$ and $x\notin U_y\}$. Then U is spg-open such that $x\notin U$ and $F\subset U$.

(iii) ⇒ (iv) Let F be any spg-closed set and $N = \cap\{G$: G is spg-open and $F\subset G\}$. Then $F\subset N \rightarrow (1)$.
Let $x\notin F$, then by (iii) $\exists\ G\in SPGO(X)$ such that $x\notin G$ and $F\subset G$.
Hence $x\notin N$ which implies $x\in N \Rightarrow x\in F$. Hence $N\subset F \rightarrow (2)$.
Therefore from (1) and (2), each spg-closed set $F = \cap\{G$: G is spg-open and $F\subset G\}$

(iv) ⇒ (v) obvious.

(v) ⇒ (vi) Let $x\notin F\in SPGC(X)$. Then X-F = G is a spg-open set containing x. Then by (v), G can be expressed as the union of spg-closed sets A contained in G, and so there is an $M\in SPGC(X)$ such that $x\in M\subset G$; and hence spgcl$\{x\}\subset G$ which implies spgcl$\{x\}\cap F = \phi$.

(vi) ⇒ (i) Let $x\in G\in SPGO(X)$. Then $x\notin(X\text{-}G)$, which is a spg-closed set. Therefore by (vi) spgcl$\{x\}\cap(X\text{-}G) = \phi$, which implies that spgcl$\{x\}\subseteq G$. Thus X is spgR_0 space.

Theorem 5.5: Let *f*: X → Y be a spg-closed one-one function. If X is weakly spg-R_0, then so is Y.

Theorem 5.6: If X is weakly spg-R_0, then for every space Y, X× Y is weakly spg-R_0.

Proof: $\cap$ spgcl$\{(x,y)\}\subseteq\cap\{$spgcl$\{x\}\times$ spgcl$\{y\}\} = \cap$ [spgcl{x}]× [spgcl{y}] $\subseteq \phi\times Y = \phi$. Hence X× Y is spg$R_0$.

Corollary 5.1:
(i) If X and Y are weakly spgR_0, then X× Y is weakly spgR_0.
(ii) If X and Y are (weakly-)R_0, then X× Y is weakly spgR_0.
(iii)If Xand Y are spgR_0, then X× Y is weakly spgR_0.
(iv) If X is spgR_0 and Y are weakly R_0, then X× Y is weakly spgR_0.

Theorem 5.7: X is spgR_0 iff for any x, $y\in X$, spgcl{x} ≠ spgcl{y}⇒ spgcl$\{x\}\cap$ spgcl$\{y\} = \phi$.

Proof: Let X is spgR_0 and x, $y\in X$ such that spgcl{x} ≠ spgcl{y} .Then $\exists\ z\in$ spgcl{x} such that $z\notin$ spgcl{y} (or $z\in$ spgcl{y}) such that $z\notin$ spgcl{x}. There exists $V\in SPGO(X)$ such that $y\notin V$ and $z\in V$; hence $x\in V$. Therefore, $x\notin$ spgcl{y}. Thus $x\in[\text{spgcl}\{y\}]^c\in SPGO(X)$, which implies spgcl$\{x\}\subset[\text{spgcl}\{y\}]^c$ and spgcl$\{x\}\cap$ spgcl$\{y\} = \phi$. The proof for otherwise is similar.

Sufficiency: Let $x\in V\in SPGO(X)$. We show that spgcl$\{x\}\subset V$. Let $y\notin V$, i.e., $y\in V^c$. Then x ≠ y and $x\notin$ spgcl{y}. Hence spgcl{x} ≠ spgcl{y}. But spgcl$\{x\}\cap$ spgcl$\{y\} = \phi$. Hence $y\notin$ spgcl{x}. Therefore spgcl$\{x\}\subset$ V.

Theorem 5.8: X is spgR_0 iff for any points x, $y\in X$, $Ker_{\{spg\}}\{x\} \neq Ker_{\{spg\}}\{y\} \Rightarrow Ker_{\{spg\}}\{x\}\cap Ker_{\{spg\}}\{y\} = \phi$.

Proof: Suppose X is spgR_0. Thus by Lemma 5.3 for any x, $y\in X$ if $Ker_{\{spg\}}\{x\} \neq Ker_{\{spg\}}\{y\}$ then spgcl{x} ≠ spgcl{y}. Assume that $z\in Ker_{\{spg\}}\{x\}\cap Ker_{\{spg\}}\{y\}$. By $z\in Ker_{\{spg\}}\{x\}$ and Lemma 5.2, it follows that $x\in$ spgcl{z}. Since $x\in$ spgcl{z}, spgcl{x} = spgcl{z}. Similarly, we have spgcl{y} = spgcl{z} = spgcl{x}. This is a contradiction. Therefore, we have $Ker_{\{spg\}}\{x\} \cap Ker_{\{spg\}}\{y\} = \phi$.

Conversely, let x, $y\in X$, s.t. spgcl{x} ≠ spgcl{y}, then by Lemma 5.3, $Ker_{\{spg\}}\{x\} \neq Ker_{\{spg\}}\{y\}$. Hence by hypothesis $Ker_{\{spg\}}\{x\}\cap Ker_{\{spg\}}\{y\} = \phi$ which implies spgcl$\{x\}\cap$spgcl$\{y\} = \phi$ Because $z\in$ spgcl{x} implies that $x\in Ker_{\{spg\}}\{z\}$ and therefore $Ker_{\{spg\}}\{x\}\cap Ker_{\{spg\}}\{z\} \neq \phi$ Therefore by Theorem 5.7 X is a spgR_0 space.

Theorem 5.9: The following properties are equivalent:
(i) X is a spg-R_0 space.
(ii) For any A ≠ ϕ and $G\in SPGO(X)$ such that $A\cap G \neq \phi$ $\exists\ F\in SPGC(X)$such that $A\cap F \neq \phi$ and $F\subset G$.

Proof: (i) ⇒ (ii): Let A ≠ ϕ and $G\in SPGO(X)$ such that $A\cap G \neq \phi$. There exists $x\in A\cap G$. Since $x\in G\in SPGO(X)$, spgcl$\{x\}\subset G$. Set F = spgcl{x}, then $F\in SPGC(X)$, $F\subset G$ and $A\cap F \neq \phi$

(ii) ⇒ (i): Let $G\in$ SPGO(X) and $x\in$ G. By (2), spgcl$\{x\}\subset$ G. Hence X is spg-R_0.

Theorem 5.10: The following properties are equivalent:
(i) X is a spg-R_0 space;
(ii) $x\in$ spgcl{y} iff $y\in$ spgcl{x}, for any points x and y in X.

Proof: (i) ⇒ (ii): Assume X is spgR_0. Let $x\in$ spgcl{y} and D be any spg-open set such that $y\in D$. Now by hypothesis, $x\in D$. Therefore, every spg-open set which contain y contains x. Hence $y\in$ spgcl{x}.

(ii) ⇒ (i): Let U be a spg-open set and x∈U. If y∉ U, then x∉spgcl{y} and hence y∉spgcl{x}. This implies that spgcl{x}⊂U. Hence X is $spgR_0$.

Theorem 5.11: The following properties are equivalent:
(i) X is a $spgR_0$ space;
(ii) If F is spg-closed, then F = $Ker_{\{spg\}}$(F);
(iii) If F is spg-closed and x∈ F, then $Ker_{\{spg\}}$\{x\}⊆F;
(iv) If x∈ X, then $Ker_{\{spg\}}$\{x\}⊂spgcl{x}.
Proof: (i) ⇒ (ii): Let x∉F∈SPGC(X) ⇒ (X-F)∈SPGO(X) and contains x. For X is $spgR_0$, spgcl({x})⊂(X-F). Thus spgcl({x})∩ F = ϕ and x∉$Ker_{\{spg\}}$(F). Hence $Ker_{\{spg\}}$(F) = F.
(ii) ⇒ (iii): A⊂B ⇒ $Ker_{\{spg\}}$(A)⊂$Ker_{\{spg\}}$(B). Therefore, by (2) $Ker_{\{spg\}}$\{x\}⊂ $Ker_{\{spg\}}$(F) = F.
(iii) ⇒ (iv): Since x∈spgcl{x} and spgcl{x} is spg-closed, by (3) $Ker_{\{spg\}}$\{x\}⊂ spgcl{x}.
(iv) ⇒ (i): Let x∈spgcl{y}. Then by Lemma 5.2 y∈$Ker_{\{spg\}}$\{x\}. Since x∈spgcl{x} and spgcl{x} is spg-closed, by (iv) we obtain y∈$Ker_{\{spg\}}$\{x\}⊆spgcl{x}. Therefore x∈spgcl{y} implies y∈spgcl{x}. The converse is obvious and X is $spgR_0$.

Corollary 5.2: The following properties are equivalent:
(i) X is $spgR_0$.
(ii) spgcl{x} = $Ker_{\{spg\}}$\{x\}∀ x∈ X.
Proof: Straight forward from Theorems 5.4 and 5.11.

Recall that a filterbase F is called spg-convergent to a point x in X, if for any spg-open set U of X containing x, there exists B∈ F such that B⊂ U.

Lemma 5.4: Let x and y be any two points in X such that every net in X spg-converging to y spg-converges to x. Then x∈ spgcl{y}.
Proof: Suppose that x_n = y for each n∈N. Then $\{x_n\}_{n\in N}$ is a net in spgcl{({y})}. Since $\{x_n\}_{n\in N}$ spg-converges to y, then $\{x_n\}_{n\in N}$ spg-converges to x and this implies that x∈spgcl{y}.

Theorem 5.12: The following statements are equivalent:
(i) X is a $spgR_0$ space;
(ii) If x, y∈X, then y∈spgcl{x} iff every net in X spg-converging to y spg-converges to x.
Proof: (i) ⇒ (ii): Let x, y∈X such that y∈spgcl{x}. Suppose that $\{x_\alpha\}_{\alpha\in I}$ is a net in X such that $\{x_\alpha\}_{\alpha\in I}$ spg-converges to y. Since y∈spgcl{x}, by Thm. 5.7 we have spgcl{x} = spgcl{y}. Therefore x∈spgcl{y}. This means that $\{x_\alpha\}_{\alpha\in I}$ spg-converges to x.
Conversely, let x, y∈ X such that every net in X spg-converging to y spg-converges to x. Then x∈ spgcl{y}[by 5.4]. By Thm. 5.7, we have spgcl{x} = spgcl{y}. Therefore y∈spgcl{x}.
(ii) ⇒ (i): Let x, y∈X such that spgcl{x}∩ spgcl{y} ≠ ϕ. Let z∈spgcl{x}∩ spgcl{y}. So ∃ a net $\{x_\alpha\}_{\alpha\in I}$ in spgcl{x} such that $\{x_\alpha\}_{\alpha\in I}$ spg-converges to z. Since z∈spgcl{y}, then $\{x_\alpha\}_{\alpha\in I}$ spg-converges to y. It follows that y∈spgcl{x}. Similarly we obtain x∈spgcl{y}. Therefore spgcl{x} = spgcl{y}. Hence X is $spgR_0$.

Theorem 5.13:
(i) Every subspace of $spgR_1$ space is again $spgR_1$.
(ii)Product of any two $spgR_1$ spaces is again $spgR_1$.

Theorem 5.14: X is $spgR_1$ iff given x ≠ y∈ X, spgcl{x} ≠ spgcl{y}.

Theorem 5.15: Every spg_2 space is $spgR_1$.

The converse is not true. However, we have the following result.

Theorem 5.16: Every spg_1 and $spgR_1$ space is spg_2.
Proof: Let x ≠ y∈X. Since X is spg_1; {x} and {y} are spg-closed sets such that spgcl{x} ≠ spgcl{y}. Since X is $spgR_1$, there exists disjoint spg-open sets U and V such that x∈U; y∈V. Hence X is spg_2.

Corollary 5.3: X is spg_2 iff it is $spgR_1$ and spg_1.

Theorem 5.17: The following are equivalent
(i) X is spg-R_1.
(ii) ∩spgcl{x} = {x}.
(iii)For any x∈ X, intersection of all spg-neighborhoods of x is {x}.
Proof: (i) ⇒ (ii) Let y ≠ x∈X such that y∈spgcl{x}. Since X is $spgR_1$, ∃ U∈SPGO(X) such that y∈U, x∉U or x∈U, y∉U. In either case y∉spgcl{x}. Hence ∩spgcl{x} = {x}.
(ii) ⇒ (iii) If y ≠ x∈X, then x∉∩spgcl{y}, so there is a spg-open set containing x but not y. Therefore y does not belong to the intersection of all spg-neighborhoods of x. Hence intersection of all spg-neighborhoods of x is {x}.
(iii) ⇒ (i) Let x ≠ y∈X. by hypothesis, y does not belong to the intersection of all spg-neighborhoods of x and x does not belong to the intersection of all spg-neighborhoods of y, which implies spgcl{x} ≠ spgcl{y}. Hence X is spg-R_1.

Theorem 5.18: The following are equivalent:
(i) X is spg-R_1.
(ii) For each pair x, y∈X with spgcl{x} ≠ spgcl{y}, ∃ a spg-open, spg-closed set V s.t. x∈V and y∉V, and

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(iii)For each pair x, y∈X with spgcl{x} ≠ spgcl{y}, ∃ *f*: X→[0, 1] s.t. *f*(x) = 0 and *f*(y) = 1 and *f* is spg-continuous.

Proof: (i) ⇒ (ii) Let x, y∈X with spgcl{x} ≠ spgcl{y}, ∃ disjoint U; W∈SPGO(X) such that spgcl{x}⊂U and spgcl{y}⊂W and V = spgcl(U) is spg-open and spg-closed such that x∈V and y∉V.

(ii) ⇒ (iii) Let x, y∈ X with spgcl{x} ≠ spgcl{y}, and let V be spg-open and spg-closed such that x∈V and y∉V. Then *f*: X→ [0, 1] defined by *f*(z) = 0 if z∈V and *f*(z) = 1 if z∉V satisfied the desired properties.

(iii) ⇒ (i) Let x, y∈ X such that spgcl{x} ≠ spgcl{y}, let *f*: X→ [0, 1] such that *f* is spg-continuous, *f*(x) = 0 and *f*(y) = 1. Then U = f^{-1}([0, 1/2)) and V = f^{-1}((1/2, 1]) are disjoint spg-open and spg-closed sets in X, such that spgcl{x}⊂U and spgcl{y}⊂V.

Theorem 5.19: If X is spg-R_1, then X is spg-R_0.

Proof: Let x∈U∈SPGO(X). If y∉U, then spgcl{x} ≠ spgcl{y}. Hence, ∃ a spg-open V such that spgcl{y}⊂V and x∉V ⇒ y∉spgcl{x}. Thus spgcl{x}⊂U. Therefore X is spg-R_0.

Theorem 5.20: X is spg-R_1 iff for x, y∈ X, $Ker_{\{spg\}}\{x\} \neq Ker_{\{spg\}}\{y\}$, ∃ disjoint U; V∈SPGO(X) such that spgcl{x}⊂U and spgcl{y}⊂V.

Spg-C_i and spg-D_i spaces, i = 0,1,2

Definition 6.1: X is said to be a

(i) spg-C_0 space if for each pair of distinct points x, y of X there exists a spg-open set G whose closure contains either of the point x or y.

(ii) spg-C_1 space if for each pair of distinct points x, y of X there exists a spg-open set G whose closure containing x but not y and a spg-open set H whose closure containing y but not x.

(iii)spg-C_2 space if for each pair of distinct points x, y of X there exists disjoint spg-open sets G and H such that G containing x but not y and H containing y but not x.

Note: spg-C_2 ⇒ spg-C_1 ⇒ spg-C_0. Converse need not be true in general as shown by the following Example.

Example 6.1:

(i) Let X = {a, b, c} and τ = {ϕ, {b}, {a, c}, X} then X is spg-C_i, i = 1, 2.

(ii) Let X = {a, b, c} and τ = {ϕ, {a}, X} then X is not spg-C_i, i = 0, 1, 2.

Theorem 6.1:

(i) Every subspace of spg-C_i space is spg-C_i.

(ii) Every spg_i spaces is spg-C_i.

(iii)Product of spg-C_i spaces are spg-C_i.

Theorem 6.2: Let (X, τ) be any spg-C_i space and A be any non empty subset of X then A is spg-C_i iff (A, $\tau_{/A}$) is spg-C_i.

Theorem 6.3: (i) If X is spg-C_1 then each singleton set is spg-closed.

(ii)In an spg-C_1 space disjoint points of X has disjoint spg- closures.

Definition 6.2: A⊂ X is called a spg-Difference(Shortly spgD-set) set if there are two U, V∈ SPGO(X) such that U ≠ X and A = U-V.

Clearly every spg-open set U different from X is a spgD-set if A = U and V = ϕ.

Definition 6.3: X is said to be a

(i) spg-D_0 if for any pair of distinct points x and y of X there exist a spgD-set in X containing x but not y or a spgD-set in X containing y but not x.

(ii) spg-D_1 if for any pair of distinct points x and y in X there exist a spgD-set of X containing x but not y and a spgD-set in X containing y but not x.

(iii)spg-D_2 if for any pair of distinct points x and y of X there exists disjoint spgD-sets G and H in X containing x and y respectively.

Example 6.2: Let X = {a, b, c} and τ = {ϕ, {b}, {a, c}, X} then X is $spgD_i$, i = 0, 1, 2.

Remark 6.2: (i) If X is rT_i, then it is spg_i, i = 0, 1, 2 and converse is false.

(ii) If X is spg_i, then it is $spg_{\{i-1\}}$, i = 1, 2.

(iii) If X is spg_i, then it is spg-D_i , i = 0, 1, 2.

(iv) If X is spg-D_i, then it is spg-$D_{\{i-1\}}$, i = 1, 2.

Theorem 6.4: The following statements are true:

(i) X is spg-D_0 iff it is spg_0.

(ii) X is spg-D_1 iff it is spg-D_2.

Corollary 6.1: If X is spg-D_1, then it is spg_0.

Proof: Remark 6.1(iv) and Theorem 6.2(i)

Definition 6.4: A point x∈X which has X as the unique spg-neighborhood is called spg.c.c point.

Theorem 6.5: For an spg_0 space X the following are equivalent:

(i) X is spg-D_1;

(ii) X has no spg.c.c point.

Proof: (i) ⇒ (ii) Since X is spg-D_1, then each point x of X is contained in a spgD-set O = U - V and thus in U. By Definition U ≠ X. This implies that x is not a spg.c.c point.
(ii) ⇒ (i) If X is spg_0, then for each x ≠ y∈ X, at least one of them, x (say) has a spg-neighborhood U containing x and not y. Thus U which is different from X is a spgD-set. If X has no spg.c.c point, then y is not a spg.c.c point. This means that there exists a spg-neighborhood V of y such that V ≠ X. Thus y∈ (V-(U) but not x and V-U is a spgD-set. Hence X is spg-D_1.

Corollary 6.2: A spg_0 space X is spg-D_1 iff there is a unique spg.c.c point in X.
Proof: Only uniqueness is sufficient to prove. If x_{00} and y_0 are two spg.c.c points in X then since X is spg_0, at least one of x_0 and y_0 say x_0, has a spg-neighborhood U such that x_0∈ U and y_0∉ U, hence U ≠ X, x_0 is not a spg.c.c point, a contradiction.

Remark 6.2: It is clear that an spg_0 space X is not spg-D_1 iff there is a unique spg-c.c point in X. It is unique because if x and y are both spg.c.c point in X, then at least one of them say x has a spg-neighborhood U containing x but not y. But this is a contradiction since U ≠ X.

Definition 6.5: X is spg-symmetric if for x and y in X, x ∈ spgcl{y} implies y ∈ spgcl{x}.

Theorem 6.6: X is spg-symmetric iff {x} is spgg-closed for each x∈ X.
Proof: Assume that x∈spgcl{y} but y∉spgcl{x}. This means that $[spgcl\{x\}]^c$ contains y. This implies that spgcl{y}⊂ $[spgcl\{x\}]^c$. Now $[spgcl\{x\}]^c$ contains x which is a contradiction.
Conversely, suppose that {x}⊂ E∈SPGO(X) but spgcl{x}⊄E. This means that spgcl{x} and E^c are not disjoint. Let y belongs to their intersection. Now we have x∈spgcl{y}⊂E^c and x∉ E. But this is a contradiction.

Corollary 6.3: If X is a spg_1, then it is spg-symmetric.
Proof: In a spg_1 space, singleton sets are spg-closed (Theorem 2.2(ii)) and therefore spg-closed (Remark 6.3). By Theorem 6.6, the space is spg-symmetric.

Corollary 6.4: The following are equivalent:
(i) X is spg-symmetric and spg_0
(ii) X is spg_1.
Proof: By Corollary 6.3 and Remark 6.1 it suffices to prove only (i) ⇒ (ii). Let x ≠ y and by spg_0, we may assume that x∈G_1⊂$\{y\}^c$ for some G_1∈SPGO(X).Then x∉spgcl{y} and hence y∉spgcl{x}. There exists a G_2∈SPGO(X) such that y∈G_2 ⊂$\{x\}^c$ and X is a spg_1 space.

Theorem 6.7: For a spg-symmetric space X the following are equivalent:
(i) X is spg_0; (ii) X is spg-D_1; (iii) X is spg_1.
Proof: (i) ⇒ (iii) Corollary 6.4 and **(iii) ⇒ (ii) ⇒ (i)** Remark 6.1.

Theorem 6.8: If *f:* X→ Y is a spg-irresolute surjective function and E is a spgD-set in Y, then the inverse image of E is a spgD-set in X.
Proof: Let E be a spgD-set in Y. Then there are spg-open sets U_1 and U_2 in Y such that E = U_1 - U_2 and U_1 ≠ Y. By the spg-irresoluteness of *f*, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are spg-open in X. Since U_1 ≠ Y, we have $f^{-1}(U_1)$ ≠ X Hence $f^{-1}(E) = f^{-1}(U_1)$-$f^{-1}(U_2)$ is a spg-D-set.

Theorem 6.9: If Y is spg-D_1 and *f:* X → Y is spg-irresolute and bijective, then X is spg-D_1.
Proof: Suppose that Y is a spg-D_1 space. Let x and y be any pair of distinct points in X. Since *f* is injective and Y is spg-D_1, there exist spg-D-sets G_z and G of Y containing *f*(X) and *f*(y) respectively, such that *f*(y) ∉G_z and *f*(X) ∉ G. By Theorem 6.8, $f^{-1}(G_z)$ and $f^{-1}(G)$ are spg-D-sets in X containing x and y, respectively. This implies that X is a spg-D_1 space.

Theorem 6.10: X is spg-D_1 iff for each pair of distinct points x, y in X there exist a spg-irresolute surjective function *f:* X→ Y, where Y is a spg-D_1 space such that *f*(x) and *f*(y) are distinct.
Proof: Necessity. For every x ≠ y∈ X, it suffices to take the identity function on X.
Sufficiency. Let x and y be any pair of distinct points in X. By hypothesis, there exists a spg-irresolute, surjective function *f* of a space X onto a spg-D_1 space Y such that *f*(x) ≠ *f*(y). Therefore, there exist disjoint spg-D-sets G_x; G_y⊂ Y such that *f*(x)∈G_x and *f*(y)∈G. Since *f* is spg-irresolute and surjective, by Theorem 6.8, $f^{-1}(G_x)$ and $f^{-1}(G)$ are disjoint spg-D-sets in X containing x and y respectively. Therefore X is spg-D_1 space.

Corollary 6.5: Let $\{X_\alpha/\alpha\in I\}$ be any family of topological spaces. If X_α is spg-D_1 for each α∈I, then the product Π X_α is spg-D_1.
Proof: Let (x_α) and (y_α) be any pair of distinct points in ΠX_α. Then there exists an index β∈I s.t. $x_\beta \neq y_\beta$. The natural projection P_β: $\Pi X_\alpha \to X_\beta$ is almost continuous and almost open and $P_\beta((x_\alpha)) = P_\beta((y_\alpha))$. Since X_β is spg-D_1, ΠX_α is spg-D_1.

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